Technical Notes

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Slender Wing in Ground Effect

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Introduction

RECENTLY, advances in computational fluid dynamics have enabled researchers to compute ground-effect solutions with complex geometries, and the inclusion of the effects of compressibility and viscosity. Examples include Katz[1] (panel method), Deese and Agarwal[2] (Euler equations), and Hashiguchi, Ohta, and Kuwahara[3] (Navier-Stokes equations). There is always a place in the literature, however, for analytic solutions that are valid for limiting values of the physical parameters, such as the wall clearance. For incompressible, irrotational flow, solutions for small values of wall clearance have been obtained by Tuck[4] for airfoils and Newman[5] for slender wings. Solutions for large values of wall clearance have been obtained by Plotkin and Kennell[6] for airfoils and Plotkin and Tan[7] for large-aspect ratio wings. It is the purpose of this Note to determine analytically the lift coefficient for a slender (low-aspect ratio) wing in ground effect for large values of wall clearance. A panel method computation will be made for comparison.

Analysis

Consider the inviscid incompressible irrotational flow of a uniform stream of speed U at angle of attack α past a thin flat slender wing of chord c, span b, and aspect ratio A. A Cartesian coordinate system is introduced with x along the stream direction and x-z a wing plane of symmetry. A ground plane parallel to the x-y plane is introduced a distance $h(x)$ below the wing. A side view is shown in Fig. 1.

For $A \ll 1$ and $\alpha \ll 1$, slender-wing theory is applicable, and the mathematical problem for the perturbation velocity potential in a local crossflow plane is

$$\phi_{yy} + \phi_{zz} = 0 \tag{1}$$

$$\phi_z(x,y,0\pm) = -U\alpha, \qquad -s \le y \le s \tag{2}$$

$$\phi_z(x,y,-h) = 0 \tag{3}$$

$$\nabla\phi \to 0 \quad \text{for} \quad y^2 + z^2 \to \infty \tag{4}$$

where $s(y)$ is the local semispan. A solution that automatically satisfies Eq. (1) and (4) is given by a distribution of vortices along the y axis from $-s$ to s with strength $\gamma(y)$ per unit length and zero total circulation. Equation (3) is satisfied by including the images of the wing vortices in the ground plane at $z = -2h$. Equation (2) then becomes an integral equation for γ and can be written as

$$\int_{-s}^{s} \gamma(y_0) K(y - y_0) \mathrm{d}y_0 = 2\pi\alpha U \tag{5}$$

where

$$K(y) = y^{-1} - y/(y^2 + 4h^2) \tag{6}$$

This integral equation is identical to the airfoil angle of attack problem in Ref. 6, and for $h/b \gg 1$, the solution proceeds as follows. Let

$$K(y) = \frac{1}{y} + h^{-1} \sum_0^\infty K_n \left(\frac{y}{h}\right)^n \tag{7}$$

and

$$\gamma(y) = \sum_0^\infty h^{-n} \gamma_n(y) \tag{8}$$

where K_n is nonzero for n odd and is given by

$$K_n = (-1)^{(n+1)/2} 2^{-(n+1)} \tag{9}$$

Equations (7–9) are substituted into Eq. (5), and terms with like powers of h^{-1} are collected. The following system of equations for the unknown $\gamma_n(y)$ is obtained:

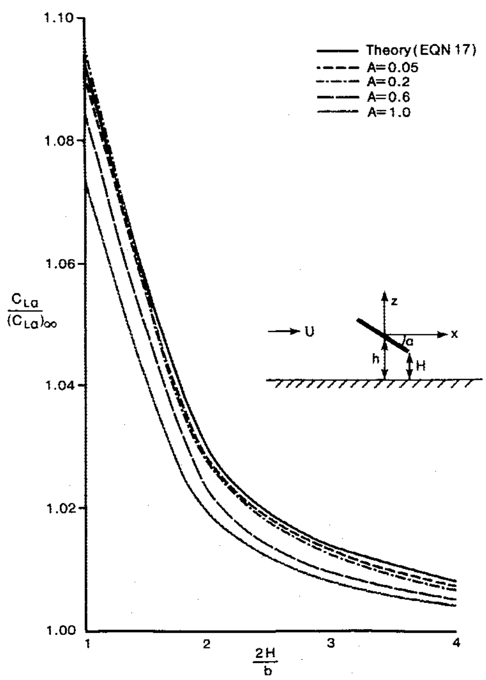

Fig. 1 Lift curve slope for slender delta wing in ground effect.

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$$\int_{-s}^{s} \frac{\gamma_0(y_0)\mathrm{d}y_0}{y-y_0} = 2\pi\alpha U \tag{10}$$

$$\int_{-s}^{s} \frac{\gamma_n(y_0)\mathrm{d}y_0}{y-y_0} = -\sum_{m=0}^{n-1} K_m \int_{-s}^{s} (y-y_0)^n \gamma_{n-m-1}(y_0)\mathrm{d}y_0$$

$$\equiv f_n(y) n \geq 1$$

Each of the $\gamma_n(y)$ has zero circulation, and the solution is (see Cheng and Rott[8])

$$\gamma_n(y) = \frac{1}{\pi^2}(s^2-y^2)^{-\frac{1}{2}} \int_{-s}^{s} \frac{f_n(y_0)(s^2-y_0^2)^{\frac{1}{2}}}{y_0-y}\mathrm{d}y_0 \tag{11}$$

The singular integrals in Eqs. (10) and (11) are to be considered in the Cauchy principal value sense. Note that γ_0 is the solution due to R.T. Jones for $h \to \infty$ (see Plotkin[9]):

$$\gamma_0(y) = -2U\alpha y(s^2-y^2)^{-\frac{1}{2}} \tag{12}$$

Results and Discussion

Once $\gamma(y)$ is obtained, the jump in velocity potential across the wing surface can be found from an integration of

$$\phi_y(x,y,0\pm) = \pm\gamma(y)/2 \tag{13}$$

With the use of the linearized Bernoulli equation, the wing lift is given by[9]

$$L = \rho U \int_{-s}^{s} [\phi(c,y,0+) - \phi(c,y,0-)]\mathrm{d}y \tag{14}$$

where ρ is the density. The nondimensional lift coefficient is

$$C_L \equiv 2L/\rho U^2 S \equiv C_{L\alpha}\alpha \tag{15}$$

where S is the planform area. Note that for $h \to \infty$ we get

$$(C_{L\alpha})_\infty = \pi A/2 \tag{16}$$

The lift curve slope for the slender wing in ground effect is

$$C_{L\alpha}/(C_{L\alpha})_\infty = 1 + 1/8(b/2H)^2 - 1/32(b/2H)^4$$
$$+ 0/(b/2H)^6] \tag{17}$$

where H is h at the wing trailing edge and additional terms can be obtained in a straightforward manner. Note that the result is independent of aspect ratio and is expected to be an increasingly accurate approximation as $A \to 0$.

To assess the validity of the slender-wing result, a widely used panel code[10] (VSAERO) was used to compute the lift coefficient for a delta wing in ground effect for values of aspect ratio less than one. (For thin wings, the method is the same as a quadrilateral vortex lattice method.) The method models the nonplanar trailing vortex sheet and allows for wake relaxation and rollup. Attached flow was assumed, and the angle of attack was 1 deg. The vortex lattice is formed by streamwise lines with cosine spacing along the span and radial lines from the tips with equal spacing along the chord. Forty panels along the chord and 10 panels along the semispan were used to deal with the small aspect ratios of interest.

Results of the computation for aspect ratios 1.0, 0.6, 0.2, and 0.05 are compared to the analytical result [Eq. (17)] in Fig. 1. It is seen that the computational results approach the analytical result as the aspect ratio approaches zero.

In conclusion, a closed-form solution is obtained for the increase in lift coefficient on a slender wing due to ground effect. The result is valid for low aspect ratios and moderate to large values of the ratio of trailing edge wall clearance to wing semispan.

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Measurements of Turbulent Flow Behind a Wing-Body Junction

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Introduction

THE fluid mechanics problem of turbulent shear flow in the junction of a wing and aircraft fuselage is of significant practical and academic importance. The adverse pressure gradient caused by the wing leads to the separation of the boundary layer on the fuselage. A horseshoe vortex is formed in the separated flow region around the wing. Downstream of the wing and above the longitudinal horseshoe vortex, which is imbedded in the fuselage boundary layer, there is a shear-layer that is formed by the two merging boundary layers on the wing. The structure of the shear flow behind the wing is also influenced by the tip vortex, especially if the aspect ratio of the wing is small.

A limited amount of published experimental data exists for the wake of a wing-body junction. Nakayama and Rahai[1] report measurements in an idealized wing-body junction where both the wing and body are represented by flat surfaces and the wing is held at zero angle of attack. Velocity measurements in the wake of a wedge at zero angle of attack are described by Morrisette and Bushnell.[2] The present Note reports measurements in the turbulent shear flow behind a wing that is mounted on a flat plate at 30 deg of angle of attack. Various physical aspects of the flow are illuminated by the experimental data that include the skin-friction coefficient and pattern, the static pressure coefficient, and two mean velocity components. A detailed discussion of the experimental results is given in Özcan and Ölçmen.[3]

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